Chapter 2: Force Vectors

Scalars and vectors

	Scalar	**Vector**
Examples	Mass, Volume, Time	Force, Velocity
Characteristics	It has a magnitude	It has a magnitude and direction
Special notation used in TAM 210/211	None	Bold font or "~" under vector symbol Ex: $\boldsymbol{A}$ or $\underset{\sim}{A}$

<u>Multiplication or division of a vector by a scalar</u>

Vector addition

All vector quantities obey the parallelogram law of addition $\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{B}$

Commutative law: $\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{B} = \boldsymbol{B} + \boldsymbol{A}$

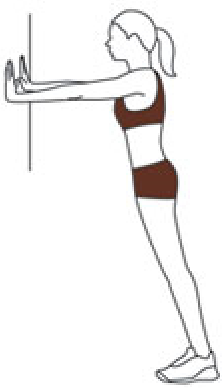
Associative law: $\boldsymbol{A} + (\boldsymbol{B} + \boldsymbol{C}) = (\boldsymbol{A} + \boldsymbol{B}) + \boldsymbol{C}$

Vector subtraction:

Scalar/Vector multiplication:

Force vectors

A force—the action of one body on another—can be treated as a vector, since forces obey all the rules that vectors do.

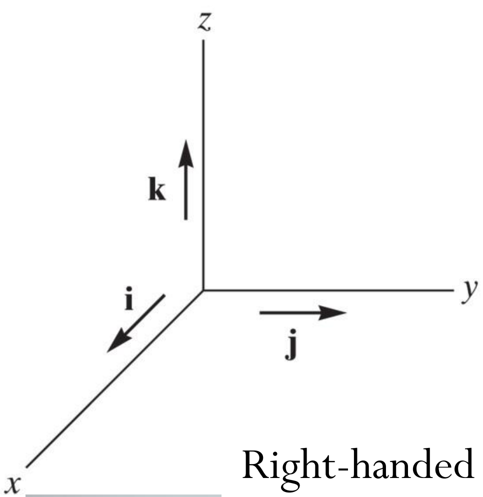


Cartesian vectors

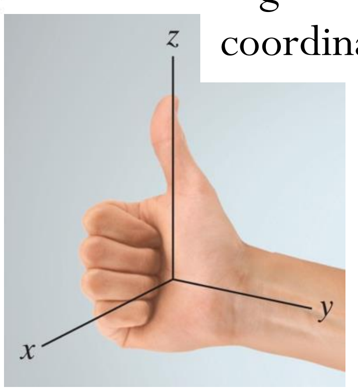
Rectangular coordinate system: formed by 3 mutually perpendicular axes, the *x*, *y*, *z* axes, with unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ in these directions.

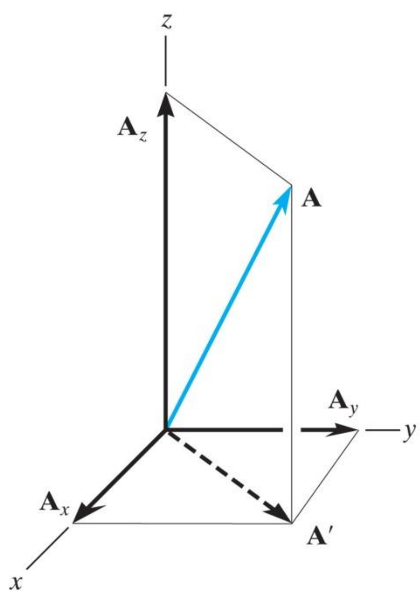
Note that we use the special notation "^" to identify *basis vectors* (instead of the "~" notation)

$$(\hat{i},\, \hat{j},\, \hat{k}) \text{ or } (\boldsymbol{i},\, \boldsymbol{j},\, \boldsymbol{k})$$



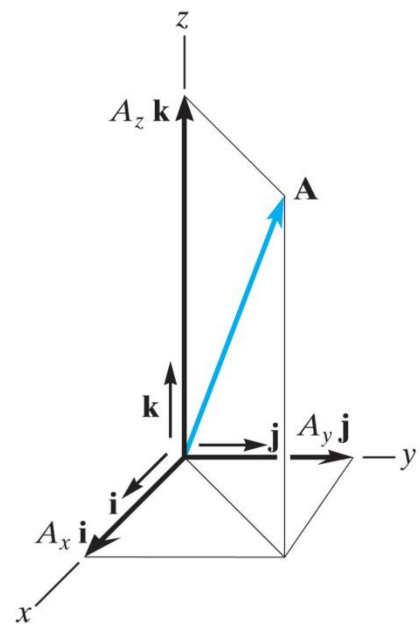
Right-handed coordinate system





Rectangular components of a vector

$$\boldsymbol{A} = \boldsymbol{A}_x + \boldsymbol{A}_y + \boldsymbol{A}_z$$

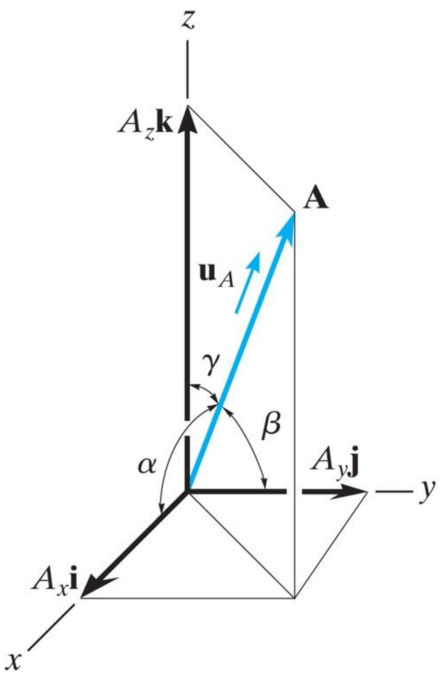


Cartesian vector representation

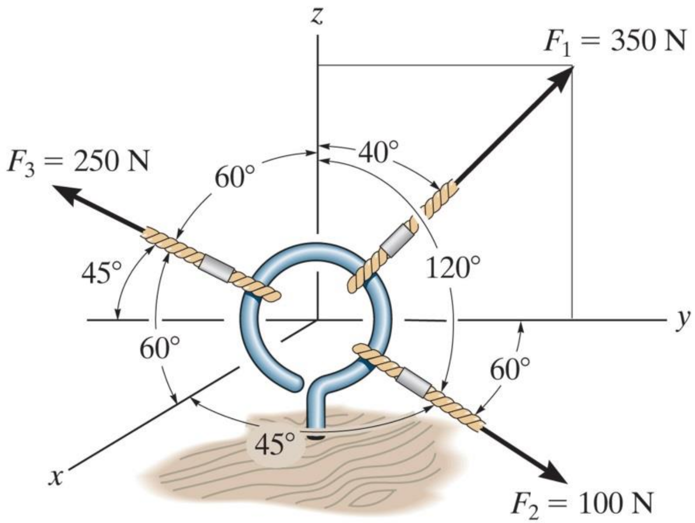
$$\boldsymbol{A} = A_x\,\boldsymbol{i} + A_y\,\boldsymbol{j} + A_z\,\boldsymbol{k}$$

Magnitude of Cartesian vectors

Direction of Cartesian vectors



Example



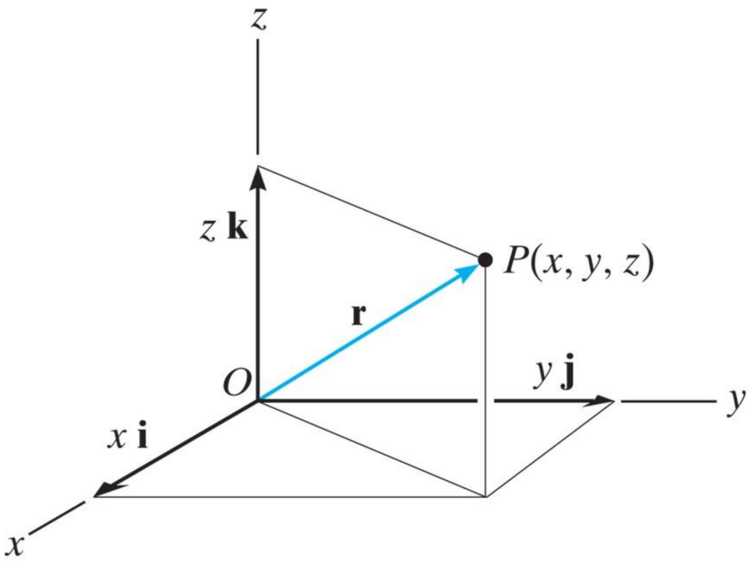
The cables attached to the screw eye are subjected to the three forces shown.

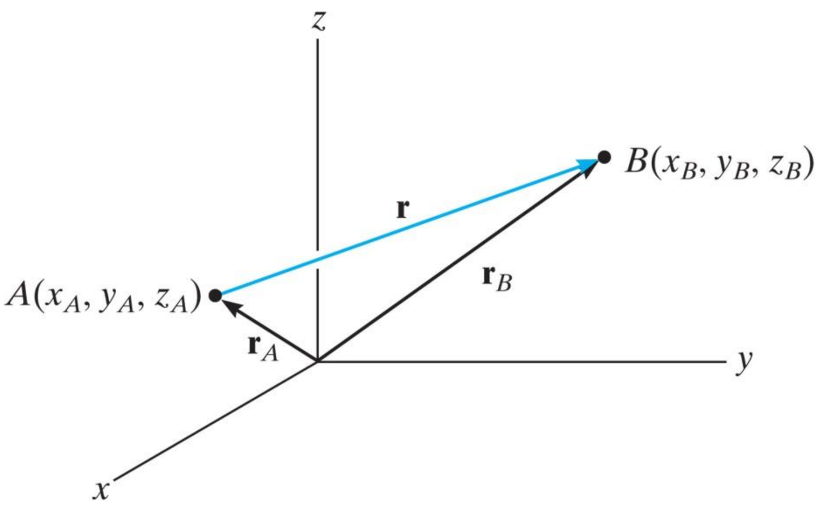
(a) Express each force vector using the Cartesian vector form (components form).

(b) Determine the magnitude of the resultant force vector

(c) Determine the direction cosines of the resultant force vector

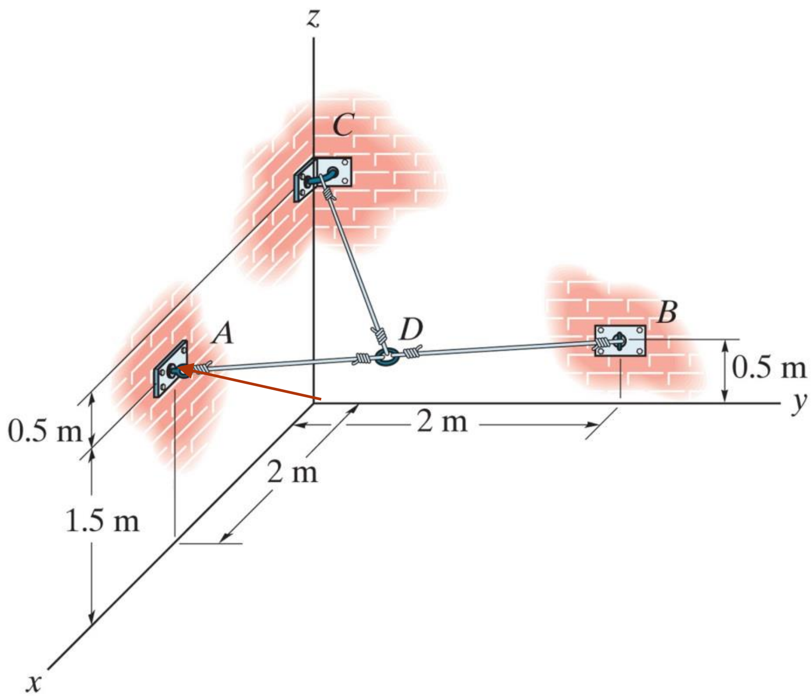
Position vectors



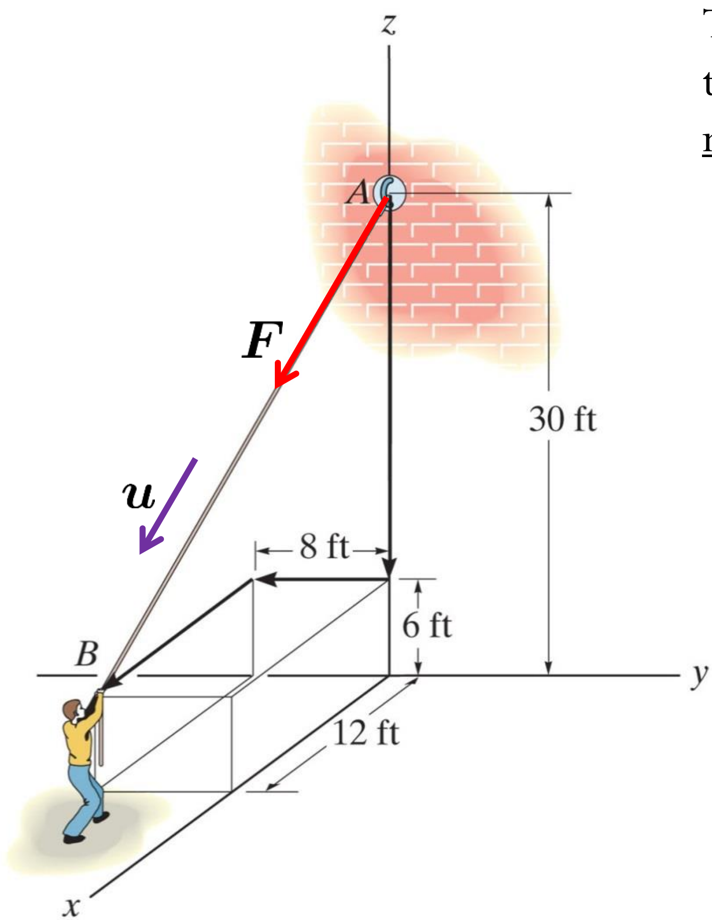


Example

The ring at D is midway between points ***A*** and ***B.*** Determine the lengths of wires AD, BD and CD



Force vector directed along a line



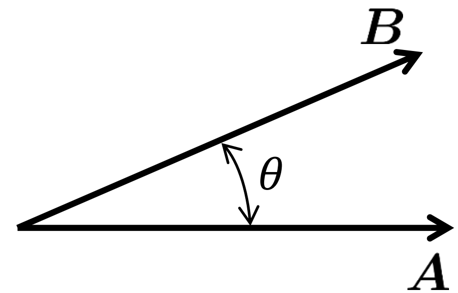
The force vector $\boldsymbol{F}$ acting a long the rope can be defined by the unit vector $\boldsymbol{u}$ (defined the direction of the rope) and the magnitude of the force.

Force vector directed along a line

Don't look up!

Dot (or scalar) product

The dot product of vectors **A** and **B** is defined as such



$$\boldsymbol{A} \cdot \boldsymbol{B} = |\boldsymbol{A}|\,|\boldsymbol{B}|\,\cos(\theta)$$

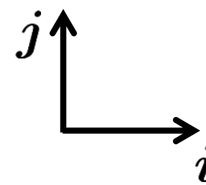
Laws of operation:

$$\boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{B} \cdot \boldsymbol{A}$$

$$\alpha(\boldsymbol{A} \cdot \boldsymbol{B}) = \alpha\boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{A} \cdot \alpha\boldsymbol{B}$$

$$\boldsymbol{A} \cdot (\boldsymbol{B} + \boldsymbol{C}) = \boldsymbol{A} \cdot \boldsymbol{B} + \boldsymbol{A} \cdot \boldsymbol{C}$$

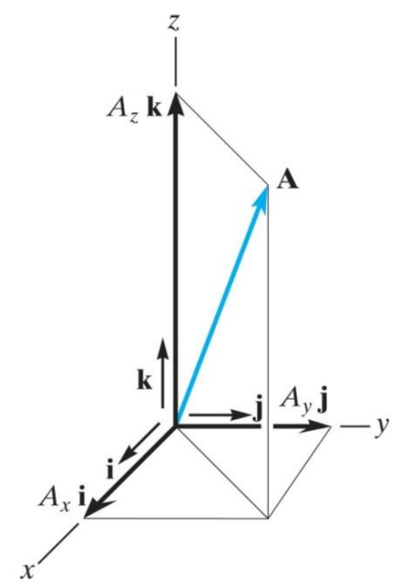
Note that:



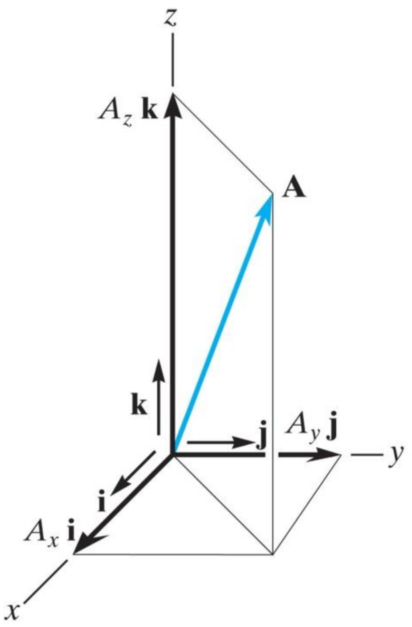
$$\boldsymbol{i} \cdot \boldsymbol{j} = 0$$

$$\boldsymbol{i} \cdot \boldsymbol{i} = 1$$

Cartesian vector formulation:



Note that:



Projections

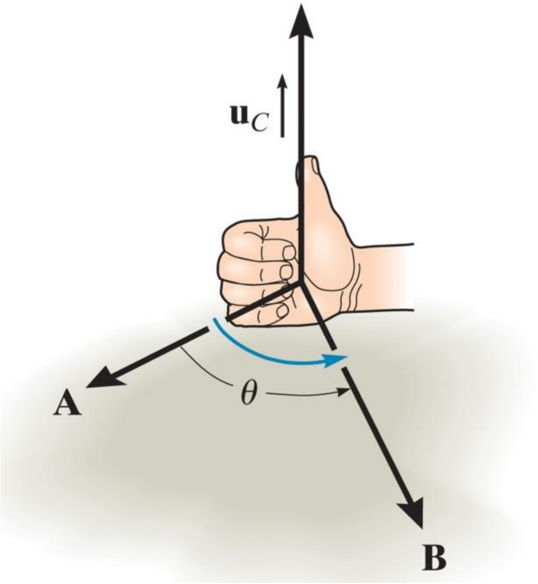
The scalar component $A_{||}$ of a vector $\boldsymbol{A}$ along (parallel to) a line with unit vector $\boldsymbol{u}$ is given by:

Example

Determine the projected component of the force vector $\boldsymbol{F}_{AC}$ along the axis of strut AO. Express your result as a Cartesian vector

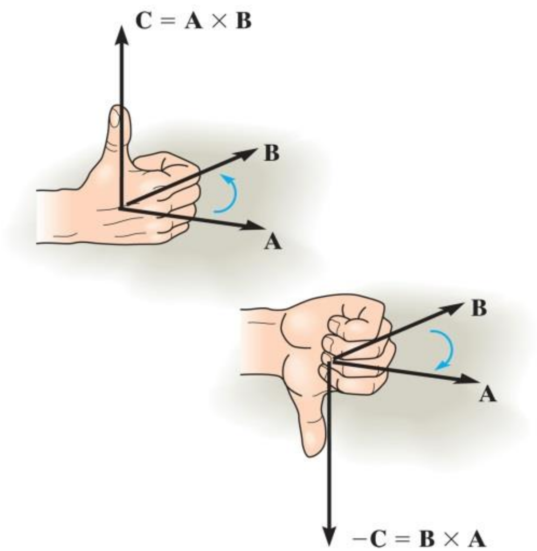
Cross (or vector) product

The cross product of vectors **A** and **B** yields the vector **C**, which is written



$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$$

Cross (or vector) product



Laws of operation:

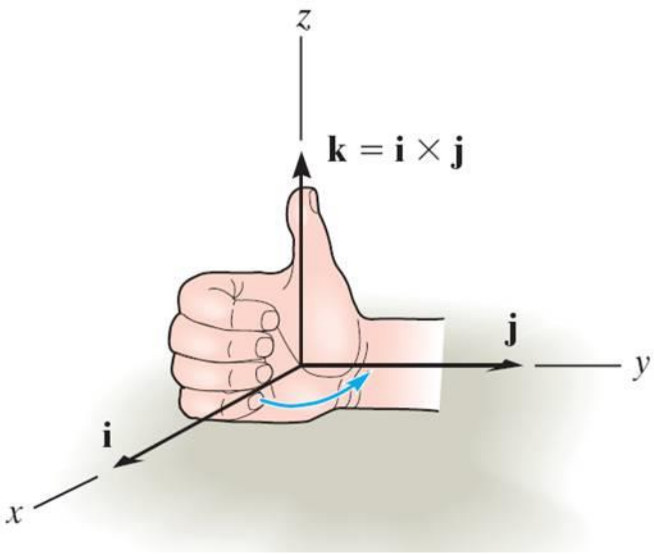
$$\boldsymbol{A} \times \boldsymbol{B} = -\boldsymbol{B} \times \boldsymbol{A}$$

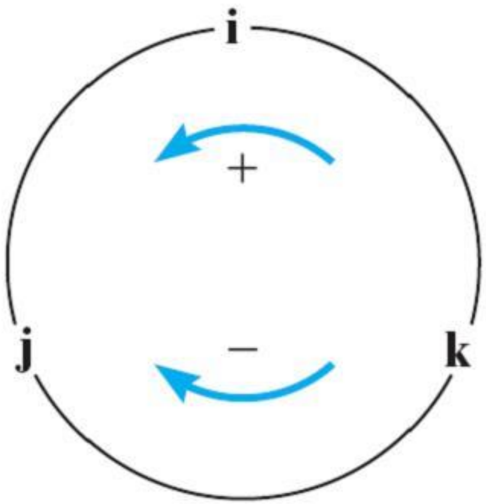
$$\alpha(\boldsymbol{A} \times \boldsymbol{B}) = (\alpha \boldsymbol{A}) \times \boldsymbol{B} = \boldsymbol{A} \times (\alpha \boldsymbol{B}) = (\boldsymbol{A} \times \boldsymbol{B})\alpha$$

$$\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{D}) = \boldsymbol{A} \times \boldsymbol{B} + \boldsymbol{A} \times \boldsymbol{D}$$

Cross (or vector) product

The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product. Note that a vector crossed into itself is zero, e.g., $i \times i = 0$



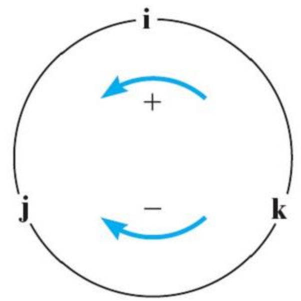


Considering the cross product in Cartesian coordinates

Cross (or vector) product

Also, the cross product can be written as a determinant.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$



Each component can be determined using 2 × 2 determinants.

Examples

Given the vectors

$$\begin{aligned} \boldsymbol{A} &= 2\,\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k} \\ \boldsymbol{B} &= 15\,\boldsymbol{i} - 20\,\boldsymbol{j} + 18\,\boldsymbol{k} \\ \boldsymbol{C} &= \boldsymbol{i} + 7\,\boldsymbol{k} \end{aligned}$$

Determine:

1. $\boldsymbol{A} + \boldsymbol{B}$
2. $\boldsymbol{B} - \boldsymbol{C}$
3. $\boldsymbol{A} \cdot \boldsymbol{B}$
4. $\boldsymbol{B} \times \boldsymbol{C}$
5. a unit vector in the direction of $\boldsymbol{C}$
6. the direction cosines of $\boldsymbol{B}$